

Power System State Estimation Using Weighted Least Squares (WLS) and Regularized Weighted Least Squares(RWLS) Method

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ABSTRACT

In this paper, a new formulation for power system state estimation is proposed. The formulation is based on regularized least squares method which uses the principle of Thikonov's regularization to overcome the limitations of conventional state estimation methods. In this approach, the mathematical unfeasibility which results from the lack of measurements in case of ill-posed problems is eliminated. This paper also deals with comparison of conventional method of state estimation and proposed formulation. A test procedure based on the variance of the estimated linearized power flows is proposed to identify the observable islands of the system. The obtained results are compared with the results obtained by conventional WLS method

Keywords – Covariance matrix, observability, power flow variance, regularized weighted least squares, weighted least squares, Thikonov regularization

I. INTRODUCTION

The aim of state estimation(SE) is to estimate the state of a power system accurately based on the various real time information and measurements available. The earliest model of state estimation was developed by Schweppe and Wildes [1]. Since then SE has been modeled into three major functions: observability analysis and restoration, state estimation and gross error detection. These three functions are usually executed separately but are related to each other and combination of these three functions determines the operation and control of the power system.

The observability analysis is basically a function that deals with the solvability of the problem of state estimation. It involves the diagnosis of whether the available set of measurements is sufficient to estimate the state of the system. Though the measurement system is planned to ensure system observability, some unpredictable situations (such as failure of the system components, malfunctioning, accidents etc.) may make the system not completely observable for different time periods there by resulting in some unobservable parts in the system [2]. The identification of these unobservable parts may help in restoration of system observability by injecting adequate pseudo measurements (restoration phase). Various numerical and topological methods have been developed for observability analysis of power system as seen in [2]-[3] and [4].

After the observability of system is ensured and solvability is verified, the state estimator provides the best estimate of system operating conditions. Most of the SE programs are formulated as an over-determined system of non-linear equations and then solved with normal equations as in [1], [5]-[6].

Finally bad data or gross error analysis is carried out to detect the measurement errors and remove/correct the gross errors in the measurements. Several bad data identification methods are based on calculation of normalized residuals or normalized Lagranges multipliers [7].

Considering the advances in state estimation so far, this paper introduces an improvement in classical WLS method of state estimation. The aim has been to develop a mathematical formulation of state estimation regardless of observability conditions. This methodology contrasts the classical methods where observability is carried out as an initial and separate analysis and eventually identifying missing measurements to restore observability before estimating the state of the system. The unfeasibility caused by lack of measurements is eliminated in this method by using regularized least squares model [8], ensuring the method is able to provide state of the system. Besides, the observability analysis is carried out by analysis of variances of the estimated parameters.

In summary, the contributions of the paper are two fold: 1) To propose a new mathematical formulation of power system state estimation which can be applied to both observable and unobservable power systems and its comparison with conventional WLS method and 2) identification of un-observable islands of the power system based on variance analysis of estimated parameters.

This paper is organized as follows: In section II, the new regularized state estimation model is presented. In section III, the procedure to find the observable islands is addressed. Proposed algorithms are presented in section IV. Tests and results with 3-bus DC, 3-bus AC, IEEE 14-bus and IEEE-30 bus system are presented in section V. Finally in section VI, conclusions are drawn.

II. REGULARIZED STATE ESTIMATOR

Consider the following measurement model:

$$z = h(x) + w \quad (1)$$

where z is an m - vector containing measurements, x is an n -vector containing the true state, $h(\cdot)$ is an m -vector of non-linear functions relating measurements to state, and w is the measurement error vector.

Assuming that measurement vectors are independent, the covariance matrix R_z , is a diagonal matrix with variances (σ_i^2) in the i th diagonal position. m is the number of measurements and n is the number of state variables. The classical state estimation using weighted least squares (WLS) formulation obtains the estimate, which minimizes the index

$$J(x) = [z - h(x)]' W [z - h(x)] \quad (2)$$

Where, $W = R_z^{-1}$. The estimate \hat{x} can be obtained only if number, the type and the location are enough to ensure the system observability.

Suppose that voltage magnitudes and voltage measurements exist in all buses. These measurements are denoted by u . In this situation, the problem becomes feasible meaning that the system is observable. Additionally, let us separate voltage measurements (real or pseudo) from rest of measurements (z) in the following way:

$$\bar{z} = \begin{bmatrix} z \\ u \end{bmatrix}, \quad \bar{h}(\hat{x}) = \begin{bmatrix} h(\hat{x}) \\ \hat{x} \end{bmatrix}, \quad \bar{W} = \begin{bmatrix} W & 0 \\ 0 & S \end{bmatrix}$$

and

$$\Delta z = \begin{bmatrix} z - h(\hat{x}) \\ u - \hat{x} \end{bmatrix}$$

where S is the diagonal weighting matrix associated to voltage measurements whose entries are inverse of the measurement variances. The above non linear problem can be solved by Gauss-Newton method which results in following iterative procedure:

$$\begin{aligned} (\bar{H}' \bar{W} \bar{H}) \Delta \hat{x}^v &= \bar{H}' \bar{W} \Delta z(\hat{x}^v) \\ \hat{x}^{v+1} &= \hat{x}^v + \Delta \hat{x}^v \end{aligned} \quad (3)$$

where,

$$\bar{H} = \frac{\partial \bar{h}(\hat{x})}{\partial \hat{x}} = [H' \quad I']'$$

and Jacobian matrix of the available measurements given by

$$H = \partial h(\hat{x}) / \partial \hat{x}$$

I_n is an identity n -matrix. The above equation can be transformed as:

$$(H' W H + S) \Delta \hat{x}^v = H' W (z - h(\hat{x}^v)) + S(u - \hat{x}^v) \quad (4)$$

This is equivalent to particular form of the multi-objective non-linear least squares problem in a weighted sum formulation:

$$\text{Min } J + S \|u - x\| \quad (5)$$

Equation (5) is known as *Thikonov Regularization* [8], [9] or simply regularizes least squares, which is employed for regularization of ill-posed problems. The diagonal weighting matrix S is known as *Thikonov* factor. By definition, S is non-singular, which makes the system represented by (4) always feasible as $(H' W H + S)$ has full rank thereby making the power system always observable.

Thus proper adjustment of S and W to minimize equation (5) will help in estimating the state of an unobservable system. For example, if the weighing factors of the real measurements are assigned by typical values and variances of pseudo measurements are considered to be small, thereby making pseudo measurements with large variances, solution can always be obtained with proposed model, although it cannot be ensured that state estimation at un-observable islands is reliable.

The main objective of the new formulation is to obtain precision of the estimated state on observable islands. On the other hand, as the estimated state of un-observable islands may be imprecise, thus these islands need to be identified accordingly which is carried out by the observability analysis proposed in next section.

III. OBSERVABILITY ANALYSIS

The above proposed algorithm solves the unfeasibility of lack of measurements in estimating the state of the system. But as the model assumes pseudo measurements with large variances at all buses without voltage measurements, it poses a new problem to identify the un-observable islands in the system.

As it is assumed that power flows with large variances are injected as pseudo measurements at all buses without voltage measurements, the estimated values at these un-observable islands may be imprecise. This perspective based on the evaluation of the variances of estimated parameters is used to identify the un-observable islands of the system.

Here, the main idea is to calculate the confidence interval of the estimated power flow. The confidence interval is a function of the standard deviation of estimated power flow on the corresponding branch and is calculated using the linearized model of power flow on branch $k-m$ by:

$$\hat{P}_{km} = (\hat{\theta}_k - \hat{\theta}_m) \frac{1}{x_{km}} = \frac{1}{x_{km}} \epsilon'_{km} \hat{\theta} \quad (6)$$

where x_{km} is the branch resistance, θ is the vector with estimated voltage angles, and ϵ_{km} is a vector with elements 1 and -1 in positions k and m , respectively. Thus an un-observable branch will result in high confidence interval as compared with observable branch.

The covariance matrix (Θ) of θ is the inverse of gain matrix (G) given as follows:

$$\Theta = (\bar{H}'_{p\theta} \bar{W}_{p\theta} \bar{H}_{p\theta})^{-1}$$

or

$$\Theta = (H'_{p\theta} W_{p\theta} H_{p\theta} + S_{p\theta})^{-1} \quad (7)$$

It can be observed that, this covariance matrix is a full ranked matrix. However, for calculation of variances of estimated power flows, only few elements are necessary. Only elements corresponding to existing branches plus diagonal elements are required. Therefore these calculations can be done effectively with sparse inverse matrix methods as proposed in [16] for bad data processing. The sparse inverse matrix can be calculated after or prior to state estimation and all observable and un-observable branches can be identified.

The variance of (6) is given by

$$\sigma_{P_{km}}^2 = \frac{1}{x_{km}^2} \epsilon'_{km} \Theta \epsilon_{km}$$

which may be represented in matrix form by

$$\sigma_{P_{km}}^2 = \frac{1}{x_{km}^2} [\Theta_{kk} + \Theta_{mm} - 2\Theta_{km}] \quad (8)$$

According to (7) and (8), the variances of the estimated power flows depend on the weighting factor. Therefore it is important to verify the behavior of these estimated variances for different measurements and pseudo measurement variance values.

IV. ALGORITHMS FOR STATE ESTIMATION AND OBSERVABILITY ANALYSIS

Algorithm 1 provides the proposed methodology for identification of observable islands. Unlike classical approaches, this can be executed before or after state estimation.

Algorithm 1: Observability Analysis

1. Calculate the covariance matrix Θ , according to (7);
2. Calculate the power flow variances for all branches σ^2 , according to (8);
3. **If** $\sigma^2 < 1$, **then**
branch $k-m$ is observable.
4. **Else**
branch $k-m$ is unobservable.
5. **end if**

Usually in cases where the network topology is symmetrical and the branches are represented by unitary reactances, it is advantageous to remove the irrelevant injections before classifying branches to eliminate their impact on results. The irrelevant injections consist of the branches which include atleast one adjacent un-observable branch.

Algorithm 2: RWLS State estimation

1. Initialize all bus voltages ($v = I$);
2. Separate voltage measurements from rest of the measurements;
3. Calculate *Thikonov* factor S ;
4. Compute the weighing matrix W ;
5. Calculate Δx^v according to equation (4);
6. **If** $\max |\Delta x^v| \leq \epsilon$ **then**
Stop. Δx^v is the estimated state.
7. **else**
update Δx^v to $x^{v+1} = x^v + \Delta x^v$;
 $v = v + I$; Back to step 5.
8. **end if**

In Algorithm 2, initially the residual ($u - x$) of pseudo measurements is set to zero to eliminate the effect of pseudo-measurements on the estimated state and also improve the convergence of the algorithm. However, if irrelevant injections are eliminated before the state estimation, zeroing the pseudo-measurement residuals is not needed.

V. TESTS AND RESULTS

In order to verify the proposed formulation, tests with 3-bus DC, 3-bus AC, IEEE 14-bus and IEEE 30-bus system have been set up.

A. 3-bus DC System

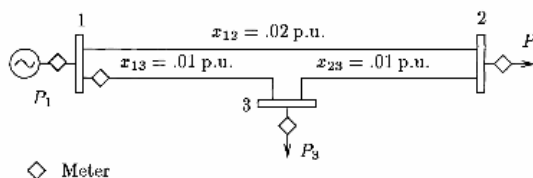


Fig.1. Three-bus DC system

The three-bus DC system of Fig.1 is used to test the proposed formulation and compare it with WLS system. The results obtained are tabulated in table I.

3 BUS DC SYSTEM				
Branch	WLS		RWLS	
	Power(pu)		Power(pu)	
	Observable	Unobservable	Observable	Unobservable
P1	4.0536	17.4225	3.807	4.0783
P2	-4.0167	-6.6905	-4.0479	-4.0693
P3	-0.0368	-10.732	-0.0152	0.2622
P13	2.036	11.3943	0.0348	0.5501
P23	NA	NA	0.0217	0.6532
P22	NA	NA	0.0644	0.5348

Table I. Comparison of WLS and RWLS with 3-bus DC system.

The obtained results show that both WLS and RWLS systems produce similar state estimates for observable system. But in case of unobservable system, WLS method of SE fails to estimate the state of system while proposed RWLS formulation produces reliable estimates.

B. 3-bus AC System

Simple three-bus AC system used for testing is shown in Fig.2.

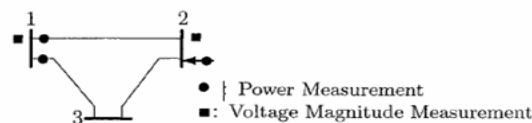


Fig.2. Three-bus AC system

The input data considered for testing the Ac system in Fig.2 is tabulated in Table II

Measurement, i	Type	Value (pu)	$\sqrt{R_{ii}}$ (pu)
1	p_{12}	0.888	0.008
2	p_{13}	1.173	0.008
3	p_2	-0.501	0.010
4	q_{12}	0.568	0.008
5	q_{13}	0.663	0.008
6	q_2	-0.286	0.010
7	V_1	1.006	0.004
8	V_2	0.968	0.004

Table II. Measurement data for 3-bus AC system

Results obtained on testing the AC system with proposed algorithm are tabulated in Table III

3 BUS AC SYSTEM							
Weighted Least Squares(WLS)				Regularized Weighted Least Squares(RWLS)			
Observable System		Unobservable System		Observable System		Unobservable System	
Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)
1	1.0259	0	0.6775	0	1.0784	0	1.7071
2	1.0011	-1.184	-0.1536	828.9733	0.9919	-0.0107	1.0135
3	0.9715	-2.598	0.5673	-7.7205	1.0002	0.0008	1.0042

Table III. Comparison of WLS and RWLS with 3-bus AC system

In this case, the system is tested by both WLS and RWLS for the available measurement data. Then an error is created in system by injecting an error measurement at bus 1-2.

Both WLS and RWLS produce almost same results in case 1. But on injection of error measurements in the system, WLS method ceases to estimate the state of the system.

C. IEEE 14-bus System

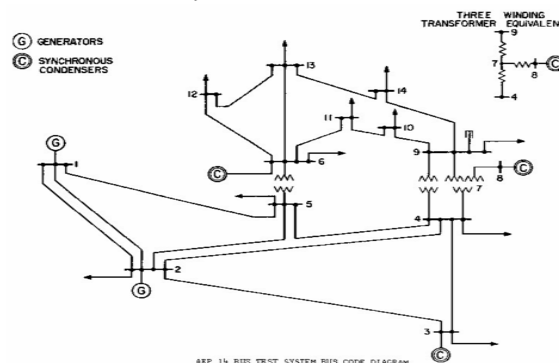


Fig.3. IEEE 14-bus system

Sample IEEE 14-bus system on which the proposed algorithm is tested to verify the accuracy in case of larger systems is shown in Fig.3.

With the available measurement data, the results obtained with proposed model are tabulated in Table IV.

14 BUS AC SYSTEM									
Weighted Least Squares(WLS)					Regularized Weighted Least Squares(RWLS)				
Observable System		Unobservable System			Observable System		Unobservable System		
Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)
1	1.006752655	0	1.88815338	0	1.06015047	0	1.060152517	0	0
2	0.989222749	-5.52651931	1.87029867	-1.681239969	1.00054376	0.14605947	1.000065684	0.133919197	0
3	0.951925553	-14.2932151	1.84507624	-4.24583971	0.99967014	-0.441945646	0.999692765	-0.447456686	0
4	0.957917275	-11.41458439	1.82473177	-4.12633056	1.000845428	-1.718780058	1.000664809	-1.728849488	0
5	0.961487141	-9.758266672	1.80739973	-2.796949139	1.000015896	2.696336552	1.000023338	2.715243445	0
6	1.016497548	-16.07983067	1.837880781	-4.743503809	1.000095808	-0.55513547	1.000120457	-0.558622204	0
7	0.991932919	-14.15104723	1.846262422	-5.021226516	1.000016127	-0.217700544	1.000039504	-0.221114637	0
8	1.028695143	-14.7499687	1.85826849	-5.0158626	0.999455699	-0.481020083	0.999473142	-0.485715391	0
9	0.976343916	-16.51254685	1.833731832	-5.506974986	1.000043297	-2.629842046	1.000062354	-2.63408383	0
10	0.975262408	-16.74764909	1.828707878	-5.426424272	1.000112099	-0.477476591	1.000123673	-0.482043403	0
11	0.95190718	-16.5396369	1.828099662	-5.957129273	1.000062573	-0.21265109	1.000070221	-0.217392959	0
12	1.000908724	-17.0202922	1.828136677	-5.028675434	0.999633673	-0.41213769	0.999637951	-0.416209111	0
13	0.993963395	-17.0583239	1.824240682	-5.03903194	1.002332034	-2.00233119	1.00236836	-2.00536835	0

Table IV. State estimation of IEEE 14-bus system

A similar approach as specified in case of 3-bus AC system by injecting error measurement has been carried out on IEEE 14-bus and IEEE 30-bus system and results are tabulated in Table IV and V respectively.

30 BUS AC SYSTEM									
Weighted Least Squares(WLS)					Regularized Weighted Least Squares(RWLS)				
Observable System		Unobservable System			Observable System		Unobservable System		
Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)	Voltage(pu)	Angle(Degrees)
1	0.98640955	0	0.84401067	0	1.05947465	0	1.05941893	0	0
2	0.970029018	-6.263481833	0.927819011	-6.371409176	0.99945596	57.28525369	0.99940488	57.28610088	0
3	0.947410829	-8.841950127	0.901218506	-9.718007924	0.99958041	57.26643197	0.99953054	57.26739471	0
4	0.938414958	-10.90214036	0.886807841	-12.18904621	0.99966421	57.29081641	0.99960965	57.29201493	0
5	0.933254951	-16.45407357	0.897862703	-17.78392917	0.99968452	57.29822694	0.9995953	57.29876589	0
6	0.93953419	-12.99747892	0.894023337	-14.45954025	0.99959526	57.30947874	0.99954121	57.31067197	0
7	0.929749062	-15.04427477	0.882503534	-16.27838524	0.99962439	57.29327857	0.99957013	57.29443847	0
8	0.944323611	-13.9607547	0.89873097	-15.2630056	0.99959778	57.28051447	0.99949377	57.28195769	0
9	0.966931641	-16.48127059	0.929778149	-19.23936026	0.99964983	57.29003631	0.99974893	57.27356691	0
10	0.947176736	-18.34454939	0.88406166	-27.4184472	0.99961591	57.29720274	0.99957445	57.2987709	0
11	1.009275509	-16.48127059	0.93425825	-19.23936026	0.99961554	57.29280209	0.99962509	57.2965919	0
12	0.974564992	-17.65175999	0.88993745	-25.45290163	0.99962913	57.29333355	0.99963866	57.29359101	0
13	0.99541598	-17.65175999	0.911788867	-25.45290163	0.99964287	57.2942437	0.99965295	57.29466588	0
14	0.955892204	-18.71367151	0.869023684	-26.81754883	0.9996506	57.30122133	0.99966544	57.3014249	0
15	0.94908715	-18.7299424	0.858021684	-27.44484875	0.9996356	57.30401355	0.99965763	57.30444452	0
16	0.95548577	-18.7299424	0.872001592	-27.55428007	0.99963591	57.30133312	0.99965504	57.30219233	0
17	0.944064788	-18.7134372	0.862363761	-26.86553905	0.99964844	57.29329	0.99969895	57.29607139	0
18	0.935172001	-19.41952484	0.836313486	-29.19273314	0.99965035	57.30047679	0.99968065	57.30092862	0
19	0.93659895	-19.60627458	0.825749566	-30.47425443	0.99964187	57.29820704	0.99967015	57.29935651	0
20	0.933877334	-19.35810739	0.833883738	-30.03154019	0.99966971	57.30197153	0.99969136	57.303495	0
21	0.932755975	-18.98208640	0.849225991	-29.01995981	0.99963529	57.301926	0.99966295	57.30419382	0
22	0.937156283	-18.7111899	0.836921117	-29.36748995	0.9996193	57.30089471	0.99964691	57.30331603	0
23	0.933112533	-18.9956521	0.843519567	-29.3049477	0.99964602	57.29856651	0.99966607	57.30070892	0
24	0.923924443	-19.07875487	0.820843268	-28.5363432	0.9996392	57.30005736	0.99969884	57.30124206	0
25	0.927015895	-18.7783836	0.853173616	-23.06480446	0.99964445	57.28617974	0.99959493	57.28651062	0
26	0.907023271	-19.2525713	0.840661741	-21.9200497	0.99929567	57.30175229	0.99925543	57.30191111	0
27	0.939492429	-18.29615761	0.873782922	-21.22005417	0.99959266	57.31684024	0.99955403	57.31686878	0

Table V. State Estimation of IEEE 30-bus system

It is observed from the above results that, as the size of the system increases, the accuracy of estimation by WLS decreases. On the other hand, RWLS method of state estimation produces reliable outputs larger systems for both observable and unobservable systems.

The test results for Algorithm 1 (observability) for 3-bus DC, 3-bus AC, IEEE 14-bus and IEEE-30 bus systems are as shown in Fig. 4, Fig.5, Fig. 6 and Fig. 7 respectively.

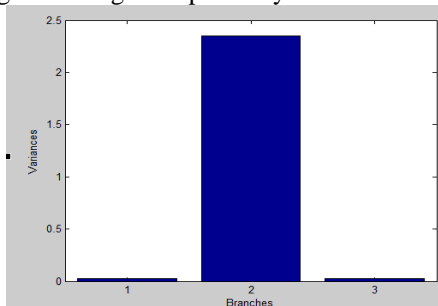


Fig.4. Three-bus DC System

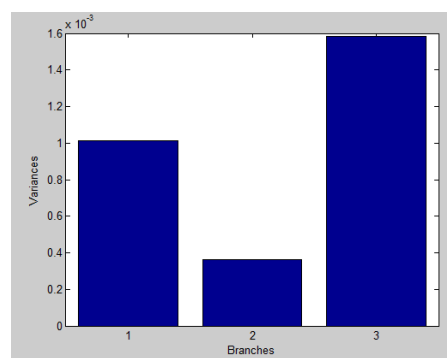


Fig.5. Three-bus AC system

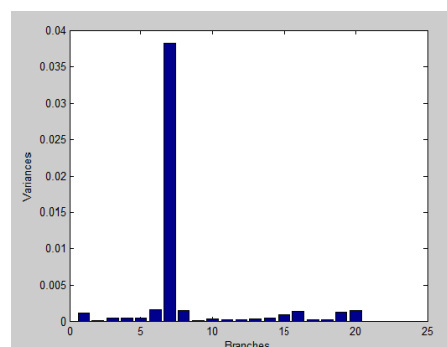


Fig.6. IEEE 14-bus system

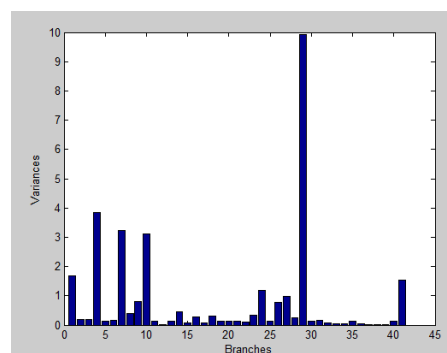


Fig.7. IEEE 30-bus system

From the above graphical representation; it can be observed that the variance of unobservable islands of the power system are greater than the threshold values and are depicted by long bar graphs. On the other hand; the observable parts of the system have variances within the threshold and are almost equal to zero.

VI. CONCLUSION

This paper has presented a new model for power system state estimation based on *Thikonov* regularization and its comparison with conventional WLS method. This new formulation has the property of regularizing the non-linear system to linear system and hence can be applied to both observable

and un-observable systems. This feature in-turn eliminates the problem of lack of measurement data which leads to ill-posed problems. A new approach of observability analysis of a power system based on variance of estimated power flows has also been formulated.

Several tests have been conducted and has been observed that SE with RWLS is far more advantageous than conventional WLS method of state estimation as it eliminates the problem of ill-posed problems, provides accurate and reliable outputs for both observable and un-observable systems, and maintains the same range of accuracy for small and large systems.

A major contribution of the proposed model is, it simplified the process of state estimation. It also transforms the problem of observability analysis from being impediment to calculation to part of problem analysis, i.e., this method eliminates the need of observability analysis before state estimation of system. Now, with this new formulation, observability analysis is only a part of system monitoring and may be carried out before of after state estimation without any impact on the estimated parameters.

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